Convexity Number Of A Degree Splitting Graph Of A Graph And Applications Of Convex Sets In Micro Cardiac Network Graph

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Abstract

The convexity number $C(G)$ of G is defined as the maximum cardinality of a proper convex set of G, that is $C(G) = max\{|S|: S$ is a convex set of G and $S \neq V(G)\}$. In this paper convexity number of a Degree splitting graph of some standard graphs are determined and application of convex sets in micro cardiac network graph is given.

Keywords: convex, convexity number, degree splitting graph, micro cardiac network graph.

Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology, we refer to [1]. A vertex v is adjacent to another vertex u if and only if there exists an edge $e = uv \in E(G)$. If $uv \in E(G)$, we say that u is a *neighbor* of v and denote by $N_G(v)$, the set of neighbors of v. A vertex v is said to be *universal vertex* if $\deg_G(v) = p - 1$. A vertex v is called an *extreme vertex* if the subgraph induced by v is complete.

A shell graph is a cycle C_p with $(p-3)$ chords sharing a common end vertex called the apex. Bistar is the graph obtained by joining the p pendent edges to both the ends of K_2. The *length* of a path is the number of its edges. Let u and v be vertices of a connected graph G. A shortest u-v path is also called a u-v *geodesic*. The (shortest path) *distance* is defined as the length of a u-v geodesic in G and is denoted by $d_G(u, v)$ or $d(u, v)$ for short if the graph is clear from the context. For a set S of vertices, let $I[S] = \bigcup_{x,y \in S} I[x, y]$. A set $S \subset V$ is called a *convex set* of G if $I[S] = S$.

These concepts were studied in[2,3].

The convexity number $C(G)$ of G is defined as the maximum cardinality of a proper convex set of G, that is $C(G) = max\{|S|: S$ is a convex set of G and $S \neq V(G)\}$

A convex set S in G with $|S| = C(G)$ is called a maximum convex set or C-set. Let $G = (V, E)$ be a graph with $V(G) = S_1 \cup S_2 \cup \dots S_n \cup T$, where S_i is the set having at least two vertices of same degree and $T = V(G) - \bigcup_{i=1}^{n} S_i$. The degree splitting graph $DS(G)$ is obtained from G by adding vertices $u_1, u_2, \dots, u_n$ and joining u_i to each vertex of S_i for $i = 1,2, \dots, n$. The micro cardiac network graph is defined in[1].

The following Theorems are used in sequel.

Theorem 1.1. Let G be a connected graph of order $p \geq 3$. Then $2 \leq C(G) \leq p - 1$.

Theorem 1.2. Let G be a connected graph of order $p \geq 3$. Then $C(G) = p - 1$ if and only if G contains a complete vertex.

Convexity Number of a degree splitting graph of a graph

Theorem 2.1. For the path $G = P_p$ $(p \geq 3)$, $C(DS(G)) = \begin{cases} 2, & p = 3 \\ p-1, & p \geq 4 \end{cases}$.

Proof. For $p = 3$, let $P_3: v_1, v_2, v_3$ be a path of order 3. Since $deg(v_1) = deg(v_3) = 1$

and $deg(v_2) = 2$, let $S_1 = \{v_1, v_3\}$ and $T = \{v_2\}$ be a two partitions of G. To obtain $DS(G)$ from G, we add a vertex $u_1$which corresponds to S_1. Therefore $|V(DS(G)| = 4$. Let $S = \{v_1, v_2\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 2$. We prove that $C(DS(G)) = 2$. On the contrary, suppose that $C(DS(G)) = 3$. Then there exists a convex set S' in $DS(G)$ such that $|S'| = 3$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = 2$.

For $p \geq 4$, let $P_p: v_1, v_2, \dots, v_p$ be a path of order p. Since $deg(v_1) = deg(v_p) = 1$

and $deg(v_i) = 2$, $2 \leq i \leq p-1$, let $S_1 = \{v_1, v_p\}$ and $S = \{v_2, \dots, v_{p-1}\}$ be a two partitions of G. To obtain $DS(G)$ from G, we add a vertices u_1 and u_2 which corresponds to S_1 and S_2 respectively. Therefore $|V(DS(G)| = p + 2$. Let $S = \{v_2, v_3, \dots, v_{p-1}, u_2\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq p - 1$. We prove that $C(DS(G)) = p - 1$. On the contrary, suppose that $C(DS(G)) \geq p$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq p$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = p - 1$.

■

Theorem 2.2. For the cycle $G = C_p$ $(p \geq 3)$, $C(DS(G)) = \begin{cases} 3, & if\ p = 3 \\ p-1, & if\ p > 4 \end{cases}$.

Proof. For $p = 3$, let $C_3: v_1, v_2, v_3, v_1$ be a cycle of order 3. Since $deg(v_i) = 2$, $1 \leq i \leq 3$, let $S_1 = \{v_1, v_2, v_3\}$. To obtain $DS(G)$ from G, we add a vertex $u_1$which corresponds to S_1. Therefore $|V(DS(G)| = 4$. Let $S = \{v_1, v_2, u_1\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 3$. By Observation 1.2, $C(DS(G)) = 3$.

For $p \geq 4$, let $C_p: v_1, v_2, \dots, v_p, v_1$ be a cycle of order p. Since $deg(v_i) = 2, 1 \leq i \leq p$, let $S_1 = \{v_1, v_2, \dots, v_p\}$. To obtain $DS(G)$ from G, we add a vertex u_1 corresponds to S_1. Therefore $|V(DS(G)| = p + 1$. Let $S = \{v_1, v_2, v_3, \dots, v_{p-2}, u_1\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq p - 1$. We prove that $C(DS(G)) = p - 1$. On the contrary, suppose that $C(DS(G)) \geq p$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq p$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = p - 1$.

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Theorem 2.3. For the complete graph $G = K_p$ $(p \geq 3)$, $C(DS(G)) = p$.

Proof. Let $K_p: v_1, v_2, \dots, v_p$ be a complete graph of order p. Since $deg(v_i) = p - 1$, $1 \leq i \leq p$, let $S_1 = \{v_1, v_2, \dots, v_p\}$. To obtain $DS(G)$ from G, we add a vertex u_1 which corresponds to S_1. Therefore $|V(DS(G)| = p + 1$. $DS(G) \cong K_{p+1}$ and by Theorem 1.2, $C(DS(G)) = p$. ■

Theorem 2.4. For the star graph $G = K_{1,p-1}$ $(p \geq 4)$, $C(DS(G)) = 2$.

Proof. Let $K_{1,p-1}: x, v_1, v_2, \dots, v_{p-1}$ be a star graph of order p. Since $deg(x) = p - 1$ and $deg(v_i) = 1$, $1 \leq i \leq p$, let $S_1 = \{v_1, v_2, \dots, v_p\}$. To obtain $DS(G)$ from G, we add a vertex u_1 which corresponds to S_1. Therefore $|V(DS(G)| = p + 1$. Let $S = \{x, v_1\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 2$. We prove that $C(DS(G)) = 2$. On the contrary, suppose that $C(DS(G)) \geq 3$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq 3$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = 2$. ■

Theorem 2.5. For the fan graph $G = F_p$ $(p \geq 4)$, $C(DS(G)) = p$.

Proof. Let $F_p: x, v_1, v_2, \dots, v_{p-1}$ be a fan graph of order p. Since $deg(x) = p - 1 = deg(v_1) = deg(v_{p-1}) = 2$ and $deg(v_i) = 3$, $2 \leq i \leq p-1$, let $S_1 = \{v_1, v_{p-1}\}$, $S_2 = \{v_2, v_3, \dots, v_{p-2}\}$ and $T = \{x\}$ be a three partitions of G. To obtain $DS(G)$ from G,

we add a vertices u_1 and u_2 which corresponds to S_1 and S_2 respectively. Therefore $|V(DS(G)| = p + 2$. Let $S = \{x, v_1, v_2, \dots, v_{p-2}, u_2\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq p$. We prove that $C(DS(G)) = p$. On the contrary, suppose that $C(DS(G)) \geq p + 1$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq p + 1$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = p$. ■

Theorem 2.6. For the wheel graph $G = W_p$ $(p \geq 4)$, $C(DS(G)) = 3$.

Proof. Let $W_p: x, v_1, v_2, \dots, v_{p-1}$ be a wheel graph of order p. Since $deg(x) = p - 1$

and $deg(v_i) = 3, 1 \leq i \leq p-1$, let $S_1 = \{v_1, v_2, \ldots, v_{p-1}\}$ and $T = \{x\}$ be a two partitions of G. To obtain $DS(G)$ from G, we add a vertices u_1 which corresponds to S_1. Therefore $|V(DS(G)| = p + 1$. Let $S = \{x, v_1, v_2\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 3$. We prove that $C(DS(G)) = 3$. On the contrary, suppose that $C(DS(G)) \geq 4$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq 4$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = 3$. ■

Theorem 2.7. For the shell graph G of order $(p \geq 5)$, $C(DS(G)) = p$.

Proof. Let $G: v_1, v_2, \ldots, v_p$ be a shell graph of order p. Since $deg(v_2) = deg(p) = 2$, $deg(v_i) = 3, 3 \leq i \leq p-1$, let $S_1 = \{v_2, v_p\}, S_2 = \{v_3, v_4, \ldots, v_{p-1}\}$ and $T = \{v_1\}$ be a three partitions of G. To obtain $DS(G)$ from G, we add a vertices u_1 and u_2 which corresponds to S_1 and S_2 respectively. Therefore $|V(DS(G)| = p + 2$. Let $S = \{v_1, v_2, v_3, v_4, \ldots, v_{p-1}, u_2\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq p$. We prove that $C(DS(G)) = p$. On the contrary, suppose that $C(DS(G)) \geq p + 1$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq p + 1$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = p$. ■

Theorem 2.8. For the comb graph $G = P_p \odot K_1$ o $(p \geq 3)$, $C(DS(G)) = 2p - 2$.

Proof. Let $G: v_1, v_2, \ldots, v_p, u_1, u_2, \ldots, u_p$ be a comb graph of order p. Since $deg(u_i) = 1$, $1 \leq i \leq p$, $deg(v_i) = de\,g(v_i) = 3, 2 \leq i \leq p-1$, let $S_1 = \{u_1. u_2, \ldots, u_p\}, S_2 = \{v_1, v_p\}$ and $S_3 = \{v_2, \ldots, v_{p-1}\}$ be a three partitions of G. To obtain $DS(G)$ from G, we add a vertices x_1, x_2 and x_3 which corresponds to S_1, S_2 and S_3 respectively. Therefore $|V(DS(G)| = 2p + 3$. Let $S = \{v_2, v_3, \ldots, v_{p-1}, u_2, u_3, \ldots, u_{p-1}, x_1, x_3\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 2p - 2$. We prove that $C(DS(G)) = 2p - 2$. On the contrary, suppose that $C(DS(G)) \geq 2p - 1$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq 2p - 1$. Then $I_{DS(G)}[S'] \neq S'$, which is a contradiction. Therefore $C(DS(G)) = 2p - 2$. ■

Theorem 2.9. For the complete bipartite graph $G = K_{r,s}$ $(r, s \geq 2)$,

$$C(DS(G)) = \begin{cases} 3, & r = s \\ 2, & r \neq s \end{cases}.$$

Proof. Let $V_1: \{v_1, v_2, \ldots, v_r\}$ and $V_2: \{u_1, u_2, \ldots, u_s\}$ be the partition of G.

Case (i) $r = s$.

In this case, each vertex of G has same degree. Therefore, we obtain $DS(G)$ from G we add a vertices x which is adjacent to every v_i and u_j. Therefore $|V(DS(G)| = r + s + 2$. Let $S = \{v_1, u_1\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 2$. We prove that $C(DS(G)) = 2$. On the contrary, suppose that $C(DS(G)) \geq 3$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq 3$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = 3$.

Case (ii) $r \neq s$.

Since $deg(v_i) = 3\ (1 \leq i \leq r)$ and $deg(u_j) = r\ (1 \leq j \leq s)$. Let $S_1 = \{v_1, v_2, \ldots, v_r\}$ and $S_2 = \{u_1, u_2, \ldots, u_i\}$ be a two partitions of G. To obtain $DS(G)$ from G we add vertices x_1 and x_2 which corresponds to S_1 and S_2 respectively. Therefore $|V(DS(G)| = r + s + 2$. Let $S = \{v_1, u_1, x\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 3$. We prove that $C(DS(G)) = 3$. On the contrary, suppose that $C(DS(G)) \geq 4$. Then there exists a convex set S' in $DS(G)$ such that $|S'| \geq 4$. Then $I_{DS(G)}[S'] = V(DS(G))$, which is a contradiction. Therefore $C(DS(G)) = 2$. ■

Theorem 2.10. For the bistar graph $G = B_{p,p}$ $(p \geq 3)$, $C(DS(G)) = 2p + 3$.

Proof. Let $G: u, v, u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p$ be bistar graph of order $2p + 2$.

Since $deg(u) = deg(v) = p$ and $deg(u_i) = deg(v_i) = 1\ (1 \leq i \leq p)$, let $S_1 = \{u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p\}$ and $S_2 = \{u, v\}$ be a two partitions of G. To obtain $DS(G)$ from G we add vertices x_1 and x_2 which corresponds to S_1 and S_2 respectively. Therefore $|V(DS(G)| = 2p + 4$. Let $S = \{u, v, u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p, x_1\}$ be a convex set of $DS(G)$ such that $C(DS(G)) \geq 2p + 3$. By Theorem 1.1, $C(DS(G)) = 2p + 3$. ■

Application of convex sets in micro cardiac network graph

A muscular organ roughly the size of a closed hand, the human heart is responsible for pumping blood throughout the body. Deoxygenated blood is brought into the body through the veins, where it is sent to the lungs for oxygenation before being pumped into the numerous arteries, which deliver nutrients and oxygen to the body's tissues by moving the blood throughout the body. The heart is situated posterior to the sternum and medial to the lungs in the thoracic cavity. The aorta, vena cava, and pulmonary arteries are all connected to the base of the heart at its better end. The diaphragm is directly above the bottom point of the heart, known as the apex. The peak of the heart, which points towards the left side, is located at the midline of the body. Since the heart is located on the left side of the body, roughly two thirds of the mass of the heart originates there, with the remaining third on the right. In [1], the various components of the human heart are depicted in Figure 3.1. The maximum cardinality of a convex set S shows the region in which the blood circulation is more in the valves connecting the heart.

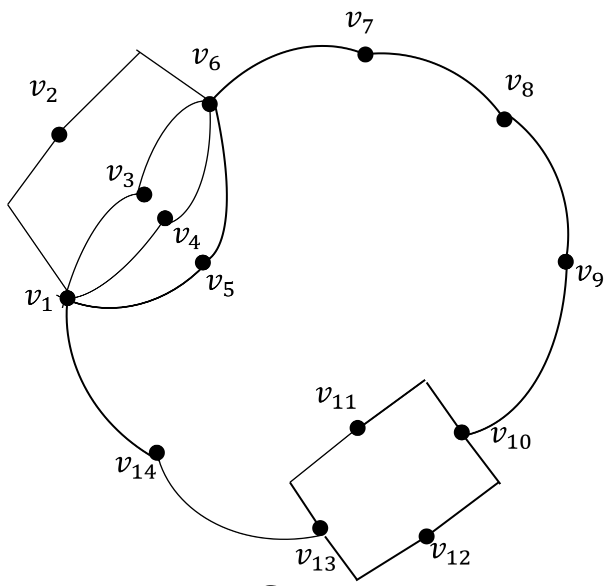

Figure 3.1
Micro Cardiac Network Graph

For the graph G given in Figure 3.1, $S = \{v_1, v_2, v_3, v_4, v_5, v_{11}, v_{12}, v_{13}, v_{14}\}$ is a maximum convex set.

Conclusion

In this article, we studied the convexity number for degree splitting graphs. Finally, we give an application of convex set in micro cardiac Network Graph.

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